

# Precoded MIMO Multiuser Scheme for Diversity Fairness

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**Abstract:** Tomlinson-Harashima precoding, design the transmit signal so that the ISI in the receiver side is very minimum. In this precoding moving the feedback filter to the transmit side to avoid error propagation problem. In this paper depth analysis of zero forcing (ZF) and minimum mean squared error (MMSE) equalizers applied to multi-input multi-output (MIMO) systems. We investigate the bit error rate performance of two types of equalizers namely ZF and MMSE for BPSK system. Zero Forcing Equalizer, applies the inverse of the channel to the received signal, to restore the signal before the channel. So this will be very useful when ISI is significant compared to noise. A minimum mean square error (MMSE) describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. MMSE equalizer, it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output.

**Keywords:** Multiple input Multiple output (MIMO) system, MMSE and ZF equalizer, Rayleigh fading channel, Tomlinson-Harashima precoder (THP).

## I. INTRODUCTION

A Multiple-Input Multiple-Output (MIMO) system is one of the most significant technical breakthroughs in modern communication system. MIMO systems are simply defined as the containing multiple transmitter antennas and multiple receiver antennas. Communication part show that MIMO systems can provide a potentially very high capacity, in many cases, grows approximately linear with the number of antennas. MIMO systems have already been implemented in wireless communication systems. Capacity limits of the Gaussian multiuser broadcast channel with multiple transmit antennas at the base station and multiple receive antennas at each user have captured a large amount of research in recent years [1].

Tomlinson-Harashima precoding (THP) was originally proposed as a pre-equalization technique for channels with inter-symbol-interference. The operation of the THP relies critically on the availability of channel state information (CSI) in order to accurately subtract the interference that otherwise would be created at each decentralized receiver. Broadcast channels have been proposed for TH precoder, including zero-forcing designs and minimum mean square error (MMSE) designs [2]. Multiple, combination of Tomlinson-Harashima precoded signals could be an efficient solution to support diversity fairness without affecting the best-ordered users [3]. In case former can be used without Channel State Information (CSI) at the transmitter and allows mitigation of fading and exploitation of transmit-receive diversity. CSI is known at the transmitter, higher throughput can be attained using spatial multiplexing, can be implemented as multibeam transmit beamforming[4]. Tomlinson-Harashima

precoding is a transmitter equalization technique where equalization is performed at the transmitter side, It can eliminate error propagation by moving the FBF of DFE to the transmitter and allow us to use current capacity-achieving channel codes[5]. THP algorithm are designed for a minimum mean square error (MMSE) approach under a constraint on the overall transmit power.

The solution to this problem requires a large number of matrix inversions (equal to the number of active users) and may be unfeasible when applied to heavy-loaded systems. All matrix inversions are replaced by a single Cholesky factorization [6]. In a noise free case, zero forcing corresponds to bringing down the ISI to zero. ISI is significant compared to noise this will be useful. Frequency response  $F(f)$  the zero forcing equalizer  $C(f)$  is constructed such that  $C(f) = 1 / F(f)$ . Combination of channel and equalizer gives a flat frequency response and linear phase  $F(f)C(f) = 1$ . Then response of a particular channel is  $H(s)$  then the input signal is multiplied by the reciprocal of this[7]. The concept behind Tomlinson-Harashima precoding is a transmitter equalization technique, elimination of the ISI of our channel and focus on ZF and MMSE to achieve better transmit diversity.

## II. TOMLINSON-HARASHIMA PRECODING

Tomlinson-Harashima Precoding, regarded as moving the feedback filter of DFE to the transmit side to avoid error propagation problem. Principle of precoding is that if transmitter knows the channel information, so we can design the transmit signal, the ISI in the receiver side is

greatly minimum. These techniques are always apply, transmitter know in advance. If channel is not fully known by the transmitter side, precoding is still correct but with a set of compromises such as residual intersymbol interference (caused by estimation errors) on transmitter removing through the linear adaptive equalization in the receiver.

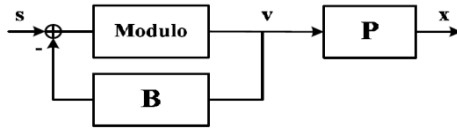


FIG. 1 TOMLINSON-HARASHIMA TRANSMITTER[2]

Tomlinson-Harashima Precoding Principle can be applied at the base station of a downlink system in which independent data symbols are transmitted to decentralized users, THP pre-subtracts the interference of previously precoded symbols that are intended for other users[2]. In case of decentralized receiver, THP operation relies critically on the availability of channel state information (CSI) in order to accurately subtract the interference on receiver. So there are different view for designing TH precoders for different channel have been proposed, including zero-forcing designs, minimum mean square error (MMSE) designs[2].

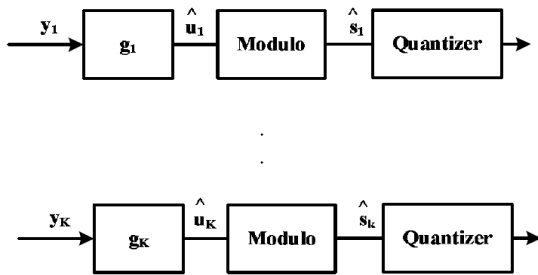


FIG. 2 DECENTRALIZED RECEIVERS [2]

On downlink base station, CSI available at the base station is generally imperfect. Systems in which each user quantizes its channel information and feeds it back to the transmitter the uncertainty in the CSI at the transmitter is mainly due to the effect of quantization errors[2]. Different result between the actual CSI and the transmitter's estimate of the CSI can result in a serious degradation of the performance of the downlink[2].

### III. MATHEMATICAL CONCEPT

Tomlinson-Harashima precoding is a transmitter equalization technique where equalization is performed at the transmitter side. Successive interference cancellation detection at receive side.

Tomlinson-Harashima structure basically two ways conceived. Left part is first way show in figure sequence is  $c[k]$  that is deducted by  $f[k]$ . The  $f[k]$  is the result of filtering the output sequence  $n[k]$  and subtracted channel impulse response by one unit in the first element[5]. Result of addition is subjected by explained modulo adder, so  $n[k]$  can be obtained.

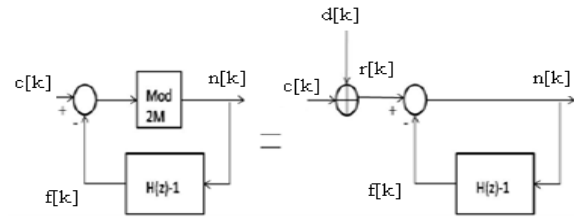


FIG. 3 TOMLINSON-HARASHIMA PRECODER AND LINEARIZED DESCRIPTION[5]

Second way to understand the TH structure is as follow: unique sequence  $s[k] \in 2MZ$  (Z as an integer number) added with data sequence  $c[k]$  and create an effective data sequence (EDS)  $r[k]$ , so  $r[k] = c[k] + d[k]$ ,  $r[k]$  is then filtered with the inverse of  $H(z)$ [5].

Below shows the mathematical relationship between the sequences:

$$n[k] = c[k] + d[k] - \sum_{k=1}^p h[k] \cdot n[k-K] = r[k] - f[k] \quad (1)$$

The values  $d[k]$  are selected symbol-by-symbol by the memory less modulo operation, which reduces  $n[k]$  to the interval  $[-M, +M]$ [5].

Linear preequalization is an extension of THP. In THP selected the current effective data symbol, this is congruent to the current  $c[k]$ , magnitude of the corresponding channel symbol  $n[k]$  is minimize[5].

Fig.4 Transmission with THP, complete scheme shows below.

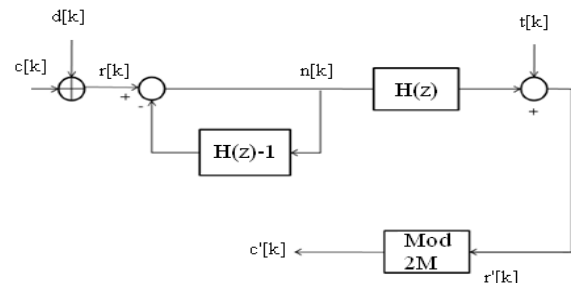


FIG.4 COMPLETE SCHEME FOR A TRANSMISSION WITH THP[5].

$$\begin{aligned} s[k] &= r[k] = \sum_{k=0}^p h[k] \cdot n[k - K] + t[k] = \\ n[k] + \sum_{k=1}^p h[k] \cdot n[k - K] + t[k] &= \\ = n[k] + f[k] + t[k] &= r[k] - f[k] + f[k] + t[k] = r[k] + t[k] \quad (2) \end{aligned}$$

Where  $t[k]$  is a white Gaussian noise sequence.

$t[k]$  easily deduced, that in noise is absence,  $t[k]$  recovered can be directly before the entry of last mod  $2M$ , so output  $t[k]$  would be reduced to range  $R = [-M, M]$  by modulo redactor without any ISI[5].

### IV. ZERO FORCING EQUALIZER

Zero Forcing Equalizer in communication systems, work on inverts the frequency response of the channel. ZF applies the inverse of the channel to the received signal, to restore the signal before the channel. The name is Zero Forcing called because, bringing down the ISI to zero in a noise free case. ISI is significant compared to noise, ZF is

very useful. Frequency response  $F(f)$  the zero forcing equalizer  $C(f)$  is constructed such that  $C(f) = 1 / F(f)$ . Channel is combined than flat frequency response and linear phase  $F(f)C(f) = 1$ .  $H(s)$  is represented channel response for a particular channel is multiplied by the reciprocal of this. Remove the effect of channel from the received signal, in particular the Inter symbol Interference (ISI)[7]. Let us consider 2x2 MIMO channel, than channel is modeled as

The received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the Second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where

$y_1, y_2$  are the received signal on the first and second antenna respectively,

$h_{1,1}$  Is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$  Is the channel from 2nd transmit antenna to 1st receive antenna,

$h_{2,1}$  Is the channel from 1st transmit antenna to 2nd receive antenna,

$h_{2,2}$  Is the channel from 2nd transmit antenna to 2nd receive antenna,

$x_1, x_2$  are the transmitted symbols and

$n_1, n_2$  are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation as follows

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Equation is

$$Y=H.x+n$$

To solve for x, we need to find a matrix  $W$  which satisfies  $WA=I$  the Zero Forcing (ZF) detector for

$$W = (AA^H)^{-1} A$$

Where  $A$  - Channel Matrix

$W$  - Equalization Matrix

Note that the off diagonal elements in the matrix  $AA^H$  are not zero, because the off diagonal elements are non zero in values. Zero forcing equalizer performing well but is not the best equalizer[7].

It is simple way and easy to implement. BPSK Modulation use Rayleigh fading channel, the BER is defined as

$$P_b = \frac{1}{2} \left( 1 - \sqrt{\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0} + 1}} \right)$$

Where

$P_b$  - Bit Error Rate

$E_b/N_0$  - Signal to noise Ratio

## V. MMSE (MINIMUM MEAN SQUARE ERROR) EQUALIZER

A minimum mean square error (MMSE) estimator performs, describes the approach which minimizes the mean square error (MSE), Common measure of estimator quality. Output of system, MMSE equalizer is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in output. Let  $x$  is define an unknown random variable, and let  $y$  is define a known random variable[7]. Measurement of  $y$  than estimator  $x^{\wedge}(y)$  is any function, mean square error is  $MSE = E\{(X^{\wedge}(y) - X)^2\}$

Expectation is taken over both  $x$  and  $y$ . An estimator achieving minimal MSE[7]. In many different cases, it is not possible to determine a much closed form for the MMSE estimator. So in these cases, one possibility is to seek the technique minimizing the MSE within a particular class, and this is a class of linear estimators. MMSE estimator is the estimator achieving minimum MSE among all estimators of the form  $AY + b$ . Measurement  $Y$  is a random vector,  $A$  is defining a matrix and  $b$  is defining a vector. Let us now try to understand the math for extracting the two symbols which interfered with each other[7]. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the Second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where

$y_1, y_2$  are the received signal on the first and second antenna respectively,

$h_{1,1}$  Is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$  Is the channel from 2nd transmit antenna to 1st receive antenna,

$h_{2,1}$  Is the channel from 1st transmit antenna to 2nd receive antenna,

$h_{2,2}$  Is the channel from 2nd transmit antenna to 2nd receive antenna,

$x_1, x_2$  are the transmitted symbols and

$n_1, n_2$  are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation as follows

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Equivalently,

$$y = H.x + n$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient  $W$  which minimizes the

$$E \{ [W(y-x)][W(y-x)]^H \}$$

To solve for  $x$ , we need to find a matrix  $W$  which satisfies  $WH = I$

Where

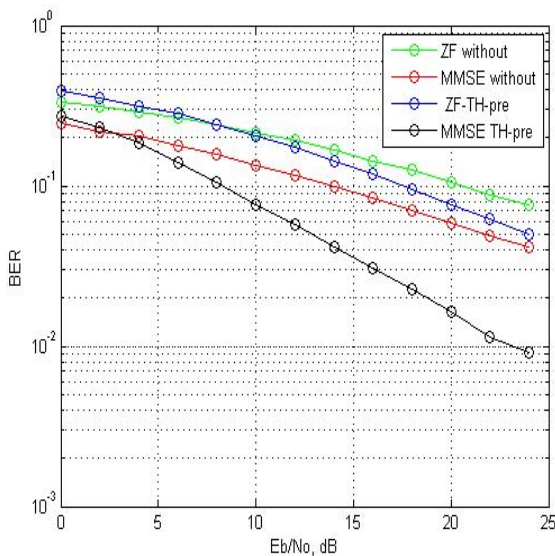
- y- Received signal
- H - Channel Matrix and
- W - Equalization Matrix
- n – Channel noise

**VI. SIMULATION RESULTS**

Tomlinson-Harashima precoding is performed for various number of transmit and receive antennas in order to achieve the better performance. Concept of Tomlinson-Harashima precoding, mainly focuses on receiver is equipped with single antenna where exists only the transmit diversity, but without any receive diversity. Zero forcing (ZF) and minimum mean square error (MMSE) algorithm is used, Zero forcing (ZF) work which invert the frequency response of the channel and (MMSE) equalizer is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output. THP is useful to achieve better transmit diversity.

**2x2 TRANSMIT AND RECEIVE ANTENNAS WITH COMPARISON**

(A) The below graph shows the performance of MIMO 2x2 ZF and MIMO 2x2 MMSE, with and without precoder two transmit antenna and two receive antennas. Here the value of bit error rate (BER) is decreases exponentially when the value of SNR increases.



**FIG.5 BER PERFORMANCE OF MIMO 2x2 ZF AND MIMO 2x2 MMSE WITH AND WITHOUT TH PRECODER**

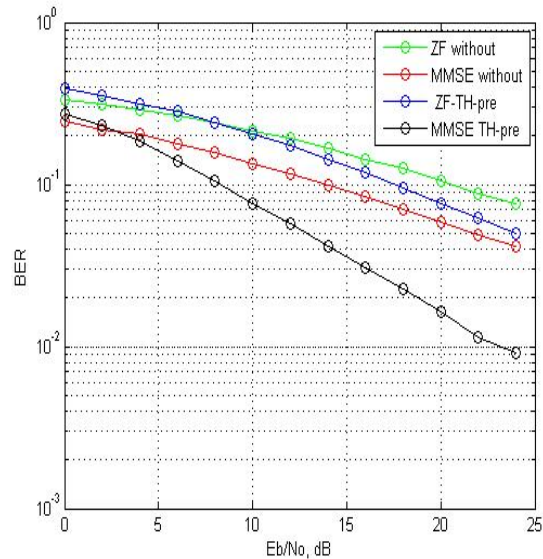
(B) The maximum value of BER is  $10^{-0.7}$  for SNR=0 db of MIMO 2x2 ZF and maximum value of MIMO 2x2 MMSE is  $10^{-0.8}$  without precoder for SNR=0 and the maximum value of BER is  $10^{-0.8}$  for SNR=0 db of MIMO 2x2 ZF and maximum value of MIMO 2x2 MMSE is  $10^{-0.8}$  with TH precoder for SNR=0 db which decreases to  $10^{-1.3}$  of MIMO 2x2 ZF and  $10^{-1.7}$  of MIMO 2x2 MMSE without precoder SNR=25 db and decreases to  $10^{-1.6}$  of MIMO 2x2 ZF and  $10^{-2.2}$  of MIMO 2x2 MMSE with TH precoder SNR=25 db.

SIGNAL-TO-NOISE RATIO (SNR)	ZF WITHOUT THP	MMSE WITHOUT THP	ZF WITH THP	MMSE WITH THP
0db	0.2953	0.2386	0.3715	0.2539
2db	0.2679	0.2133	0.3376	0.2063
4db	0.2433	0.1921	0.3049	0.1641
6db	0.2251	0.1736	0.2699	0.1259
8db	0.1966	0.1523	0.2334	0.0933
10db	0.1692	0.1286	0.2016	0.0683
12db	0.1528	0.1139	0.1664	0.0493
14db	0.1302	0.0918	0.1399	0.0365
16db	0.1060	0.0812	0.1118	0.0253
18db	0.0924	0.0660	0.0951	0.0209
20db	0.0750	0.0553	0.0746	0.0151
22db	0.0612	0.0465	0.0626	0.0107
24db	0.0522	0.0362	0.0478	0.0077

**TABLE I. COMPARISON OF BER PERFORMANCE ZF AND MMSE WITH AND WITHOUT TH PRECODER.**

**3x3 TRANSMIT AND RECEIVE ANTENNAS WITH COMPARISON**

(A) The below graph shows the performance of MIMO 3x3 ZF and MIMO 3x3 MMSE, with and without precoder three transmit antenna and three receive antennas. Here the value of bit error rate (BER) is decreases exponentially when the value of SNR increases.



**FIG.6 BER PERFORMANCE OF MIMO 3x3 ZF AND MIMO 3x3 MMSE WITH AND WITHOUT TH PRECODER**

(B) The maximum value of BER is  $10^{-0.7}$  for SNR=0 db of MIMO 3x3 ZF and maximum value of MIMO 3x3 MMSE is  $10^{-0.8}$  without precoder for SNR=0 db and the maximum value of BER is  $10^{-0.7}$  for SNR=0 db of MIMO 3x3 ZF and maximum value of MIMO 3x3 MMSE is  $10^{-0.7}$  with TH precoder for SNR=0 db which decreases to  $10^{-1.4}$  of MIMO 3x3 ZF and  $10^{-1.7}$  of MIMO 3x3 MMSE without precoder SNR=25 db and decreases to  $10^{-1.6}$  of MIMO 3x3 ZF and  $10^{-2.2}$  of MIMO 3x3 MMSE with TH precoder SNR=25 db.

SIGNAL-TO-NOISE RATIO (SNR)	ZF WITHOUT THP	MMSE WITHOUT THP	ZF WITH THP	MMSE WITH THP
0db	0.3337	0.2445	0.3899	0.2740
2db	0.3122	0.2190	0.3555	0.2324
4db	0.2908	0.2033	0.3158	0.1834
6db	0.2637	0.1779	0.2828	0.1398
8db	0.2401	0.1571	0.2400	0.1045
10db	0.2152	0.1347	0.2066	0.0753
12db	0.1924	0.1159	0.1744	0.0579
14db	0.1686	0.0983	0.1426	0.0419
16db	0.1419	0.0834	0.1195	0.0308
18db	0.1260	0.0703	0.0948	0.0226
20db	0.1048	0.0587	0.0759	0.0165
22db	0.0876	0.0484	0.0623	0.0113
24db	0.0755	0.0414	0.0495	0.0092

**TABLE II. COMPARISON OF BER PERFORMANCE ZF AND MMSE WITH AND WITHOUT TH PRECODER.**

**VII.CONCLUSION**

In this dissertation, a new precoding concept is discussed, which is called Tomlinson Harashima precoding. Some simulations are performed here to check the performance of the system. Concepts and advantages of Tomlinson Harashima, zero forcing (ZF), minimum mean square error (MMSE) are applied on the system. Tomlinson Harashima is a transmitter equalization technique, improvement on the elimination of the ISI of our channel. Zero forcing equalizer used on transmitter end and signal transmit on receiver than BER is calculate after that MMSE equalizer are used on receiver side, BER performance of ZF and MMSE is compared . In this paper mainly focus on TH precoding using ZF and MMSE to achieve better transmit diversity.

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